UE Fluids Problem F17 solution Fall '07

Velocity field of each piece:

	A	B	C
Freestream:	$u = 1 \rightarrow$	$1 \rightarrow$	$1 \rightarrow$
Left source sheet, $\lambda = +3$:	$u = -1.5 \leftarrow$	$1.5 \rightarrow$	$1.5 \rightarrow$
Right source sheet: $\lambda = -1$	$0.5 \rightarrow$	$0.5 \rightarrow$	$-0.5 \leftarrow$

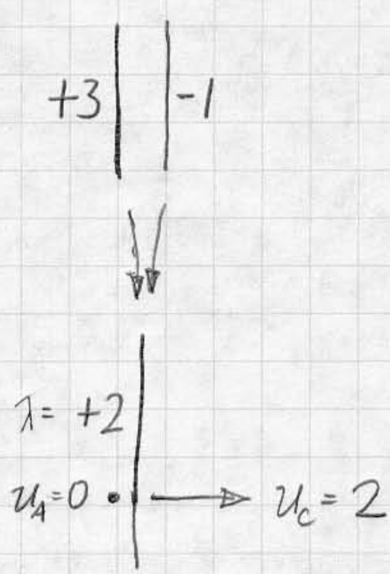
Note: Velocity jump across each sheet obeys $\Delta V_n = \lambda$

Total velocities:

Add up contributions at A, B, C:

	A	B	C
	1	1	1
	-1.5	$+1.5$	$+1.5$
	$+0.5$	$+0.5$	-0.5
$\longrightarrow$ $u =$	$\boxed{0}$	$\boxed{3}$	$\boxed{2}$

Optional additional check:



For away, the two sheets together look like one sheet of strength
$\lambda = +3 - 1 = +2$

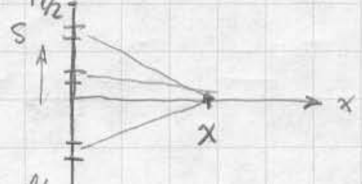
So we expect that $u_C - u_A = \Delta V_n = \lambda = 2$ ✓

UE Fluids F18 Solution Fall '07

a) Contribution of one ds piece: $d\psi = \frac{\gamma}{2\pi} \ln r \, ds$; $r = \sqrt{x^2+s^2}$

or $d\psi = \frac{\gamma}{4\pi} \ln(r^2) ds = \frac{\gamma}{4\pi} \ln(x^2+s^2)\, ds$

Superposition of all ds pieces along sheet



$$\psi = \int d\psi = \frac{\gamma}{4\pi} \int_{-l/2}^{l/2} \ln(x^2+s^2)\, ds$$

Using the given integral: $\int_{-l/2}^{l/2} \ln(s^2+x^2) ds = s \ln(s^2+x^2) - 2s + 2x \arctan\left(\frac{s}{x}\right)\Bigg|_{-l/2}^{l/2}$

$$= l/2 \ln\left[\left(\frac{l}{2}\right)^2 + x^2\right] - \left(-\frac{l}{2}\right) \ln\left[\left(\frac{l}{2}\right)^2 + x^2\right] - l$$

$$+ 2x\left[\arctan \frac{l/2}{x} - \arctan \frac{-l/2}{x}\right]$$

$$= l \ln\left[\left(\frac{l}{2}\right)^2 + x^2\right] - l + 4x \arctan\left(\frac{l/2}{x}\right)$$

$$\therefore \psi(x) = \frac{\gamma}{4\pi}\left[l \ln\left[\left(\frac{l}{2}\right)^2 + x^2\right] - l + 4x \arctan\left(\frac{l/2}{x}\right)\right]$$

cancel

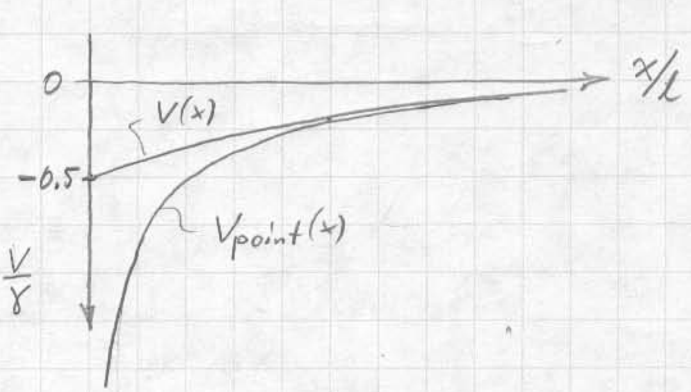
b) $$v = -\frac{\partial \psi}{\partial x} = -\frac{\gamma}{4\pi}\left[\frac{2lx}{(l/2)^2+x^2} + 4\arctan\left(\frac{l/2}{x}\right) + 4x \frac{1}{\left(\frac{l/2}{x}\right)^2+1} \cdot \frac{-l/2}{x^2}\right]$$

$$\boxed{V(x) = -\frac{\gamma}{\pi} \arctan \frac{l/2}{x}}$$

For point vortex: $\Gamma = \gamma l$, $\boxed{V_{point}(x) = \frac{-\Gamma}{2\pi x} = -\frac{\gamma l}{2\pi x}}$

c) The two curves asymptote to the same $V(x)$ as $r/l \to \infty$

This makes sense, since the sheet looks like a point from far away



UE Fluids Problem F19 + F20 solution Fall '07

a) Curvature $\kappa \approx \frac{d^2y}{dx^2}$ for shallow curves (as given, $h \ll l$)

$y = h \sin^2(\pi x/l) \qquad dy/dx = 2\pi h/l \sin(\pi x/l)\cos(\pi x/l) = \frac{\pi h}{l}\sin(2\pi x/l)$

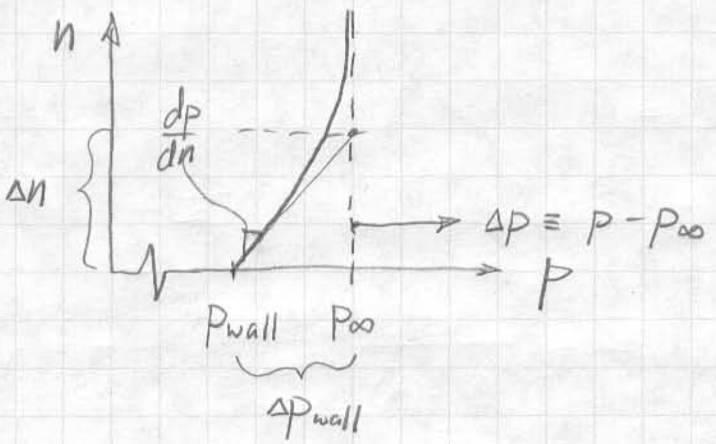
$$\frac{d^2y}{dx^2} = 2\pi^2 \frac{h}{l^2}\cos(2\pi x/l)$$

Curvature at center of bump, $x = \frac{l}{2}$: $\boxed{\kappa = \left.\frac{d^2y}{dx^2}\right|_{x=l/2} = -2\pi^2\frac{h}{l^2}}$

$$\boxed{\frac{dp}{dn} = -\rho V^2 \kappa = +\rho V_\infty^2\, 2\pi^2 \frac{h}{l^2}}$$

b) $\boxed{\Delta p_{wall} \equiv p_{wall} - p_\infty \cong \left(p_\infty - \frac{dp}{dn}\Delta n\right) - p_\infty = -\frac{dp}{dn}\Delta n = -\rho V_\infty^2\, 2\pi^2 \frac{h\,\Delta n}{l^2}}$

negative



c) $\Delta p = -\rho V_\infty^2\, 2\pi^2 \frac{h\,\Delta n}{l^2} \cong -10\left(\frac{1}{2}\rho V_\infty^2\right)\frac{h}{l}$

or $4\pi^2 \frac{h\,\Delta n}{l^2} \simeq 10\frac{h}{l}$

$$\boxed{\Delta n \simeq \frac{10}{4\pi^2}\, l = 0.25\, l}$$

d) $|\Delta C_p| \equiv \frac{|\Delta p|}{\frac{1}{2}\rho V_\infty^2} = 10\frac{h}{l} < 0.01$

Must have $\boxed{h < 0.001\, l}$

For 10 cm = 100 mm bump: $\boxed{h < 0.001 \cdot 100\,\text{mm} < 0.1\,\text{mm}}$

That's only slightly more than thickness of paper!